Problem 6. Let the population of a city in millions be given by $P = f(t) = 0.005t^3 + 2.7$, where t is years since 1982.

(a) Evaluate and interpret $f(30)$.

$f(30) = 0.005(30)^3 + 2.7$ pop. ~~34~~ in millions in 2012

(b) Find a formula for $f^{-1}(P)$.

$y = 0.005t^3 + 2.7$

$y - 2.7 = 0.005t^3$

$\frac{y-2.7}{0.005} = t^3$

$\boxed{\sqrt[3]{\frac{y-2.7}{0.005}} = t}$

(c) Evaluate and interpret $f^{-1}(30)$.

$f^{-1}(30) = \sqrt[3]{\frac{30-2.7}{0.005}}$ the number of years until we have a pop. of 30mil

Problem 7. Consider the following functions. Which are inverses of each other?

$f(x) = -6 + 4x^3$ $\quad g(x) = \sqrt[3]{\frac{x-6}{4}}$ $\quad h(x) = \sqrt[3]{\frac{x}{4}} - 6$

$p(x) = 6 + 4x^3$ $\quad q(x) = \sqrt[3]{\frac{x+6}{4}}$. $\quad r(x) = \sqrt[3]{\frac{x}{4}} + 6$

$h(x) =$
$y = \sqrt[3]{\frac{x}{4}} - 6$
$y + 6 = \sqrt[3]{\frac{x}{4}}$
$4(y+6)^3 = x$

$y = -6 + 4x^3$
$\frac{y+6}{4} = x^3 \rightarrow \sqrt[3]{\frac{y+6}{4}}$

$y = 6 + 4x^3$
$\sqrt[3]{\frac{y-6}{4}} = x$

Problem 8. In 2014, a new world record for the deepest scuba dive was set by Ahmed Gabr. Ahmed's depth, in feet, below the surface of the water during his dive may be modeled by the equation $D(t) = -9t^2 + 198t$ for $0 \leq t \leq 11$, where t is time in minutes since Ahmed entered the water.

(a) Evaluate and interpret the following:

(i) $D(11)$

D(11) is Ahmed's depth in feet after 11 min

(ii) $1089 - D(t)$

$1089 - D(t) = 9t^2 - 198t + 1089$

~~This function tells us how far~~

Don't worry about interpretation. There is none if we don't know what 1089 is.

(b) Interpret the meaning of $D^{-1}(750)$. Do not evaluate the expression.

Remember the input of D^{-1} is the output of D, so $D^{-1}(750)$ is the time it takes to get to a depth of 750 ft.

(c) Suppose the temperature of the water, $F = T(d)$, in degrees Fahrenheit, is decreasing exponentially as Ahmed descends, where d is his depth in feet below the surface of the water. In particular, $T(0) = 82$, and $T(350) = 70$.

(i) Find an exponential function modeling $T(d)$ (round your growth factor to at least four decimal places).

$$T(t) = 82b^t$$

$$70 = T(350) = 82b^{350}$$

$$\sqrt[350]{\frac{70}{82}} = b$$

$$T(t) = 82\left(\sqrt[350]{\frac{70}{82}}\right)^t$$

(ii) Try to find a formula for $T^{-1}(F)$. If you can't, explain why.

Need logs!

(d) Using your model from part (c), evaluate and interpret $T(D(6))$.

The temperature of the water after traveling for 6 minutes.